Recall:

A principal of $2000 is invested at 7%/a. compounded annually for 5 years. Find the amount of the investment at the end of the 5 year term. How much of the total is the interest earned.

$$A = P(1+i)^n$$
$$A = 2000(1+0.07)^5$$
$$A = 2000(1.07)^5$$
$$A = 2000(1.402551731)$$
$$A = 2805.103461$$
$$\therefore A = \$2805.10$$

Since the final amount (A) includes both the original amount and the interest earned we say that A = P + I

$$A = P + I$$
$$2805.10 = 2000 + I$$
$$2805.10 - 2000 = I$$
$$\therefore I = \$805.10$$

Today we want to look at the situation in which the compounding periods are less than one year.

The most commonly used compounding periods are;

- Annually (1 time per year)
- Semi-annually (2 times per year, every 6 months)
- Quarterly (4 times per year, every 3 months)
- Monthly (12 times per year)
- Semi-monthly (24 times per year) not the same as every 2 weeks
- Bi-weekly (26 times per year, every two weeks)
- Weekly (52 times per year)
- Daily (365 times per year)

For compounding periods less than one year we need to adjust both the "i" and the "n" for our formula.

Suppose an investment earns 6%/a. compounded semi-annually for 4 years.
When the interest is compounded semi-annually that means it is paid out 2 times per year or every 6 months.

Therefore the annual rate of 6%/a. is actually 3% every 6 months. Ie: $i = \frac{0.06}{2} = 0.03$

In 4 years, there are 4 x 2, or 8 compounding periods. Thus n = 8

Suppose an investment earns 9%/a. compounded monthly for 5 years.
When the interest is compounded monthly that means it is paid out 12 times during the year.

Therefore the annual rate of 9%/a. is actually $\frac{3}{4}\%$ every month. Ie: $i = \frac{0.09}{12} = 0.0075$

In 5 years, there are 5 x 12, or 60 compounding periods. Thus n = 60.

Example # 1:

A principal of $400 is invested at 5%/a. compounded semi-annually for 6 years. Determine the amount of the investment.

$A = ?$
$P = \$400$
$i = \frac{0.05}{2} = 0.025$
$n = 6 \times 2 = 12$

$A = P(1+i)^n$
$A = 400(1+0.025)^{12}$
$A = 400(1.025)^{12}$
$A = 400(1.344888824)$
$A = 537.9555297$
$\therefore A = \$537.96$

Example # 2:

Henry invested $850 at 6%/a. compounded monthly for 5 years. What is the amount of the investment at maturity?

$A = ?$
$P = \$850$
$i = \frac{0.06}{12} = 0.005$
$n = 5 \times 12 = 60$

$A = P(1+i)^n$
$A = 850(1+0.005)^{60}$
$A = 850(1.005)^{60}$
$A = 850(1.348850153)$
$A = 1146.52263$
$\therefore A = \$1146.52$

Example # 3:

A $500 GIC (Guaranteed Investment Certificate) pays $3\frac{3}{4}\%/a.$ compounded quarterly. How much interest will the GIC earn in 4.5 years?

$A = ?$
$P = \$500$
$i = \frac{0.0375}{4} = 0.009375$
$n = 4.5 \times 4 = 18$

$A = P(1+i)^n$
$A = 500(1+0.009375)^{18}$
$A = 500(1.009375)^{18}$
$A = 500(1.1828939)$
$A = 591.44695$
$\therefore A = \$591.45$

$A = P + I$
$591.45 = 500 + I$
$591.45 - 500 = I$
$\therefore I = \$91.45$